Summer Work for Students Entering Calculus I

The work in this packet is designed to take about 45-60 minutes per week, it is suggested not to leave it all for the last couple weeks of August, you won't be happy if you do!

The problems are here and the answers will be emailed by July 15. You will turn this work in the first day you have class. Please do the problems and check your answers (marking which are correct and looking for your mistakes in the incorrect ones). Please use your textbooks both from Precalculus and for the coming year as references for any trouble you find. In addition Miss Hedges will be checking email over the summer so you can email questions that you have. All topics in this packet should be review!

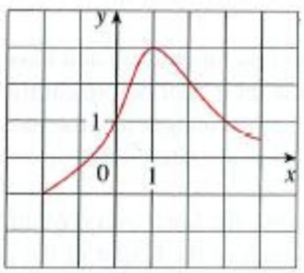
Enjoy the summer but keep your mind and skills sharp with this work!

Functions

"four ways to represent a function"

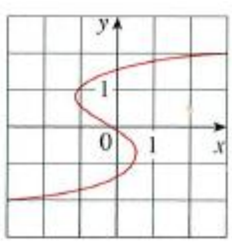
1. If $f(x) = x + \sqrt{2 - x}$ and $g(u) = u + \sqrt{2 - u}$, is it true that $f = g$?

3. The graph of a function f is given.
   (a) State the value of $f(1)$.
   (b) Estimate the value of $f(-1)$.
   (c) For what values of x is $f(x) = 1$?
   (d) Estimate the value of x such that $f(x) = 0$.
   (e) State the domain and range of f.
   (f) On what interval is f increasing?



7–10 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

7.



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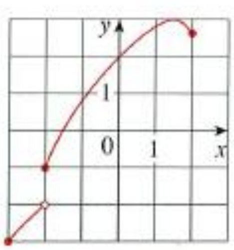


13. You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.

15. The graph shows the power consumption for a day in September in San Francisco. (P is measured in megawatts; t is measured in hours starting at midnight.)
   (a) What was the power consumption at 6 AM? At 6 PM?
   (b) When was the power consumption the lowest? When was it the highest? Do these times seem reasonable?

Pacific Gas & Electric

23. The number N (in millions) of US cellular phone subscribers is shown in the table. (Midyear estimates are given.)

t	1996	1998	2000	2002	2004	2006
N	44	69	109	141	182	233

   (a) Use the data to sketch a rough graph of N as a function of t.
   (b) Use your graph to estimate the number of cell-phone subscribers at midyear in 2001 and 2005.

25. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a + 1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a + h)$.

27–30 Evaluate the difference quotient for the given function. Simplify your answer.

$$\frac{f(x+h) - f(x)}{h}$$

27. $f(x) = 4 + 3x - x^2$, $\quad \dfrac{f(3 + h) - f(3)}{h}$

29. $f(x) = \dfrac{1}{x}$, $\quad \dfrac{f(x) - f(a)}{x - a}$

31–37 Find the domain of the function.

31. $f(x) = \dfrac{x + 4}{x^2 - 9}$

33. $f(t) = \sqrt[3]{2t - 1}$

35. $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

37. $F(p) = \sqrt{2 - \sqrt{p}}$

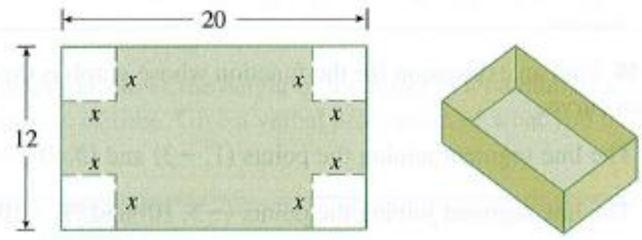
39–50 Find the domain and sketch the graph of the function.

49. $f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

51–56 Find an expression for the function whose graph is the given curve.

53. The bottom half of the parabola $x + (y - 1)^2 = 0$

63. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



73–78 Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

73. $f(x) = \dfrac{x}{x^2 + 1}$

75. $f(x) = \dfrac{x}{x + 1}$

77. $f(x) = 1 + 3x^2 - x^4$

"Mathematical Models"

1–2 Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

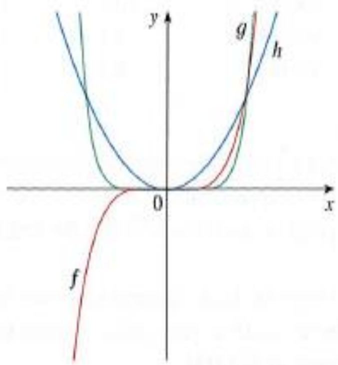
1. (a) $f(x) = \log_2 x$ (b) $g(x) = \sqrt[4]{x}$

(c) $h(x) = \dfrac{2x^3}{1 - x^2}$ (d) $u(t) = 1 - 1.1t + 2.54t^2$

(e) $v(t) = 5^t$ (f) $w(\theta) = \sin\theta\cos^2\theta$

3–4 Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator.)

3. (a) $y = x^2$ (b) $y = x^5$ (c) $y = x^8$



9. Find an expression for a cubic function f if $f(1) = 6$ and $f(-1) = f(0) = f(2) = 0$.

11. If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation $c = 0.0417D(a + 1)$. Suppose the dosage for an adult is 200 mg.

(a) Find the slope of the graph of c. What does it represent?

(b) What is the dosage for a newborn?

15. Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at 70°F and 173 chirps per minute at 80°F.

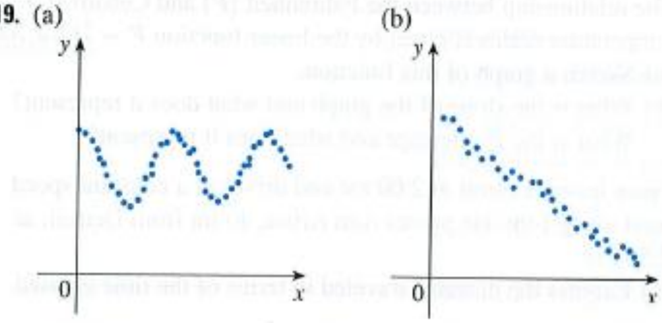
(a) Find a linear equation that models the temperature T as a function of the number of chirps per minute N.

(b) What is the slope of the graph? What does it represent?

(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

19–20 For each scatter plot, decide what type of function you might choose as a model for the data. Explain your choices.

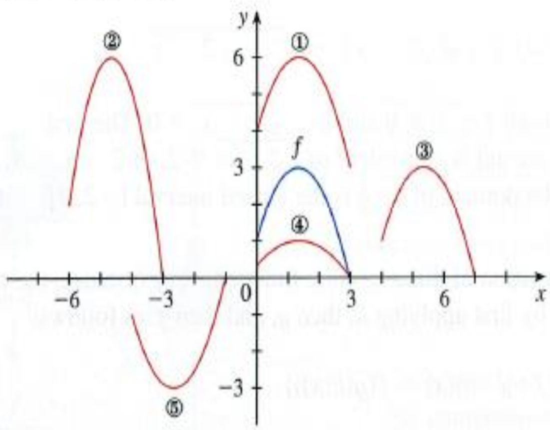
19. (a) (b)



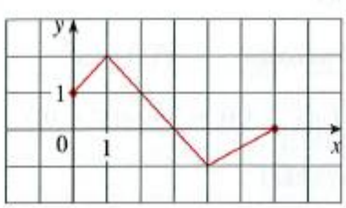
"New Functions from Old Functions"

1. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.
(a) Shift 3 units upward. (b) Shift 3 units downward.
(c) Shift 3 units to the right. (d) Shift 3 units to the left.
(e) Reflect about the x-axis. (f) Reflect about the y-axis.
(g) Stretch vertically by a factor of 3.
(h) Shrink vertically by a factor of 3.

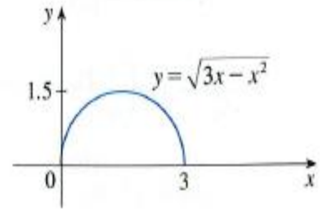
3. The graph of $y = f(x)$ is given. Match each equation with its graph and give reasons for your choices.
(a) $y = f(x - 4)$ (b) $y = f(x) + 3$
(c) $y = \frac{1}{3}f(x)$ (d) $y = -f(x + 4)$
(e) $y = 2f(x + 6)$



5. The graph of f is given. Use it to graph the following functions.
(a) $y = f(2x)$ (b) $y = f\left(\frac{1}{2}x\right)$
(c) $y = f(-x)$ (d) $y = -f(-x)$



6–7 The graph of $y = \sqrt{3x - x^2}$ is given. Use transformations to create a function whose graph is as shown.



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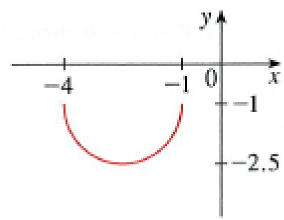


9–24 Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Section 1.2, and then applying the appropriate transformations.

9. $y = \dfrac{1}{x + 2}$

11. $y = -\sqrt[3]{x}$

13. $y = \sqrt{x - 2} - 1$

15. $y = \sin\left(\frac{1}{2}x\right)$

17. $y = \frac{1}{2}(1 - \cos x)$

19. $y = 1 - 2x - x^2$

21. $y = |x - 2|$

23. $y = \left|\sqrt{x} - 1\right|$

29–30 Find (a) $f + g$, (b) $f - g$, (c) fg, and (d) f/g and state their domains.

29. $f(x) = x^3 + 2x^2$, $g(x) = 3x^2 - 1$

31–36 Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

31. $f(x) = x^2 - 1, \quad g(x) = 2x + 1$

33. $f(x) = 1 - 3x, \quad g(x) = \cos x$

35. $f(x) = x + \dfrac{1}{x}, \quad g(x) = \dfrac{x+1}{x+2}$

37–40 Find $f \circ g \circ h$.

37. $f(x) = 3x - 2, \quad g(x) = \sin x, \quad h(x) = x^2$

39. $f(x) = \sqrt{x - 3}, \quad g(x) = x^2, \quad h(x) = x^3 + 2$

41–46 Express the function in the form $f \circ g$.

41. $F(x) = (2x + x^2)^4$

43. $F(x) = \dfrac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$

45. $v(t) = \sec(t^2)\tan(t^2)$

47–49 Express the function in the form $f \circ g \circ h$.

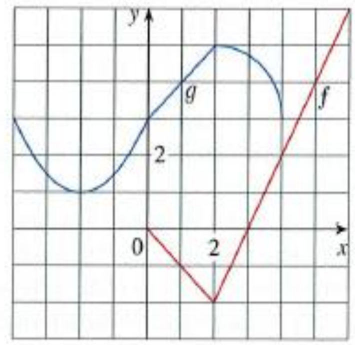
47. $R(x) = \sqrt{\sqrt{x} - 1}$

49. $H(x) = \sec^4(\sqrt{x})$

51. Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.

(a) $f(g(2))$ (b) $g(f(0))$ (c) $(f \circ g)(0)$

(d) $(g \circ f)(6)$ (e) $(g \circ g)(-2)$ (f) $(f \circ f)(4)$



53. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.

(a) Express the radius r of this circle as a function of the time t (in seconds).

(b) If A is the area of this circle as a function of the radius, find $A \circ r$ and interpret it.

55. A ship is moving at a speed of 30 km/h parallel to a straight shoreline. The ship is 6 km from shore and it passes a lighthouse at noon.

(a) Express the distance s between the lighthouse and the ship as a function of d, the distance the ship has traveled since noon; that is, find f so that $s = f(d)$.

(b) Express d as a function of t, the time elapsed since noon; that is, find g so that $d = g(t)$.

(c) Find $f \circ g$. What does this function represent?

61. (a) If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function f such that $f \circ g = h$. (Think about what operations you would have to perform on the formula for g to end up with the formula for h.)

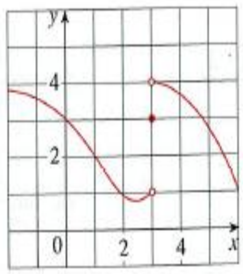
(b) If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, find a function g such that $f \circ g = h$.

"Limit of a Function"

5. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 3^-} f(x)$ (c) $\lim_{x\to 3^+} f(x)$

(d) $\lim_{x\to 3} f(x)$ (e) $f(3)$

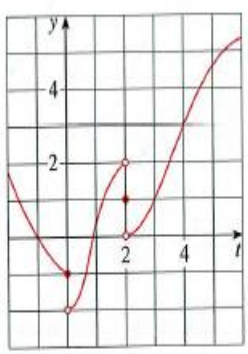


7. For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t\to 0^-} g(t)$ (b) $\lim_{t\to 0^+} g(t)$ (c) $\lim_{t\to 0} g(t)$

(d) $\lim_{t\to 2^-} g(t)$ (e) $\lim_{t\to 2^+} g(t)$ (f) $\lim_{t\to 2} g(t)$

(g) $g(2)$ (h) $\lim_{t\to 4} g(t)$

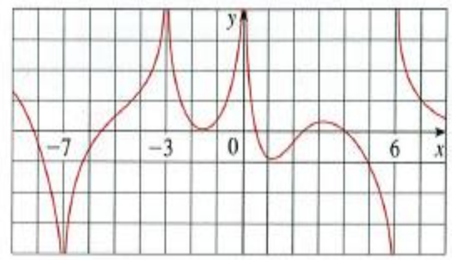


9. For the function f whose graph is shown, state the following.

(a) $\lim_{x\to -7} f(x)$ (b) $\lim_{x\to -3} f(x)$ (c) $\lim_{x\to 0} f(x)$

(d) $\lim_{x\to 6^-} f(x)$ (e) $\lim_{x\to 6^+} f(x)$

(f) The equations of the vertical asymptotes.



11–12 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

11. $f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leqslant x < 1 \\ 2 - x & \text{if } x \geqslant 1 \end{cases}$

13–14 Use the graph of the function f to state the value of each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$ (c) $\lim_{x\to 0} f(x)$

13. $f(x) = \dfrac{1}{1 + 2^{1/x}}$

15–18 Sketch the graph of an example of a function f that satisfies all of the given conditions.

15. $\lim_{x\to 0^-} f(x) = -1, \quad \lim_{x\to 0^+} f(x) = 2, \quad f(0) = 1$

23–26 Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

25. $\lim_{x\to 1} \dfrac{x^6 - 1}{x^{10} - 1}$

29–37 Determine the infinite limit.

29. $\lim_{x \to -3^+} \frac{x + 2}{x + 3}$

31. $\lim_{x \to 1} \frac{2 - x}{(x - 1)^2}$

33. $\lim_{x \to -2^+} \frac{x - 1}{x^2(x + 2)}$

35. $\lim_{x \to 2\pi^-} x \csc x$

37. $\lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6}$

"Limit Laws"

1. Given that

$$\lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 2} g(x) = -2 \qquad \lim_{x\to 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to 2} [f(x) + 5g(x)]$ (b) $\lim_{x\to 2} [g(x)]^3$

(c) $\lim_{x\to 2} \sqrt{f(x)}$ (d) $\lim_{x\to 2} \dfrac{3f(x)}{g(x)}$

(e) $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$ (f) $\lim_{x\to 2} \dfrac{g(x)h(x)}{f(x)}$

3–9 Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

3. $\lim_{x\to 3} (5x^3 - 3x^2 + x - 6)$

5. $\lim_{t\to -2} \dfrac{t^4 - 2}{2t^2 - 3t + 2}$

7. $\lim_{x\to 8} (1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$

9. $\lim_{x\to 2} \sqrt{\dfrac{2x^2 + 1}{3x - 2}}$

11–32 Evaluate the limit, if it exists.

13. $\lim_{x\to 5} \dfrac{x^2 - 5x + 6}{x - 5}$

17. $\lim_{h\to 0} \dfrac{(-5 + h)^2 - 25}{h}$

23. $\lim_{x\to -4} \dfrac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

25. $\lim_{t\to 0} \dfrac{\sqrt{1 + t} - \sqrt{1 - t}}{t}$

31. $\lim_{h\to 0} \dfrac{(x + h)^3 - x^3}{h}$

"Review"

1. (a) What is a function? What are its domain and range?
   (b) What is the graph of a function?
   (c) How can you tell whether a given curve is the graph of a function?

3. (a) What is an even function? How can you tell if a function is even by looking at its graph? Give three examples of an even function.
   (b) What is an odd function? How can you tell if a function is odd by looking at its graph? Give three examples of an odd function.

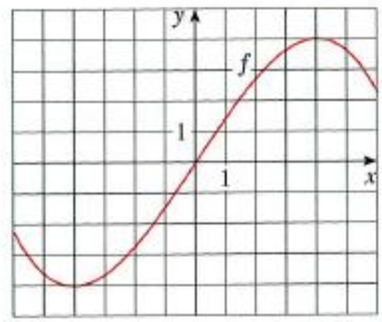
4. What is an increasing function?

8. Draw, by hand, a rough sketch of the graph of each function.
   (a) $y = \sin x$ (b) $y = \tan x$
   (c) $y = 2^x$ (d) $y = 1/x$
   (e) $y = |x|$ (f) $y = \sqrt{x}$

9. Suppose that f has domain A and g has domain B.
   (a) What is the domain of $f + g$?
   (b) What is the domain of fg?
   (c) What is the domain of f/g?

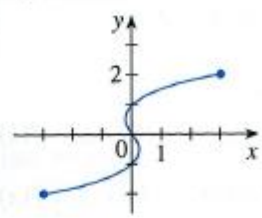
11. Suppose the graph of f is given. Write an equation for each of the graphs that are obtained from the graph of f as follows.
   (a) Shift 2 units upward.
   (b) Shift 2 units downward.
   (c) Shift 2 units to the right.
   (d) Shift 2 units to the left.
   (e) Reflect about the x-axis.
   (f) Reflect about the y-axis.
   (g) Stretch vertically by a factor of 2.
   (h) Shrink vertically by a factor of 2.
   (i) Stretch horizontally by a factor of 2.
   (j) Shrink horizontally by a factor of 2.

1. Let f be the function whose graph is given.
   (a) Estimate the value of $f(2)$.
   (b) Estimate the values of x such that $f(x) = 3$.
   (c) State the domain of f.
   (d) State the range of f.
   (e) On what interval is f increasing?
   (f) Is f even, odd, or neither even nor odd? Explain.

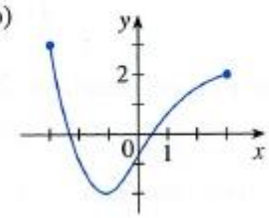


2. Determine whether each curve is the graph of a function of x. If it is, state the domain and range of the function.

(a)



(b)



3. If $f(x) = x^2 - 2x + 3$, evaluate the difference quotient

$$\frac{f(a + h) - f(a)}{h}$$

5–8 Find the domain and range of the function. Write your answer in interval notation.

5. $f(x) = 2/(3x - 1)$
7. $y = 1 + \sin x$

9. Suppose that the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f.
   (a) $y = f(x) + 8$ (b) $y = f(x + 8)$
   (c) $y = 1 + 2f(x)$ (d) $y = f(x - 2) - 2$
   (e) $y = -f(x)$ (f) $y = 3 - f(x)$

11–16 Use transformations to sketch the graph of the function.

11. $y = -\sin 2x$

13. $y = 1 + \frac{1}{2}x^3$

15. $f(x) = \dfrac{1}{x + 2}$

17. Determine whether f is even, odd, or neither even nor odd.

(a) $f(x) = 2x^5 - 3x^2 + 2$

(b) $f(x) = x^3 - x^7$

(c) $f(x) = \cos(x^2)$

(d) $f(x) = 1 + \sin x$

19. If $f(x) = \sqrt{x}$ and $g(x) = \sin x$, find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, (d) $g \circ g$, and their domains.

23. The graph of f is given.

(a) Find each limit, or explain why it does not exist.

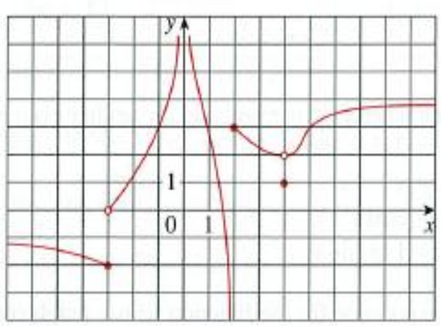
(i) $\lim\limits_{x \to 2^+} f(x)$ (ii) $\lim\limits_{x \to -3^+} f(x)$

(iii) $\lim\limits_{x \to -3} f(x)$ (iv) $\lim\limits_{x \to 4} f(x)$

(v) $\lim\limits_{x \to 0} f(x)$ (vi) $\lim\limits_{x \to 2^-} f(x)$

(b) State the equations of the vertical asymptotes.

(c) At what numbers is f discontinuous? Explain.



25–38 Find the limit.

25. $\lim\limits_{x \to 0} \cos(x + \sin x)$

27. $\lim\limits_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

29. $\lim\limits_{h \to 0} \dfrac{(h - 1)^3 + 1}{h}$

31. $\lim\limits_{r \to 9} \dfrac{\sqrt{r}}{(r - 9)^4}$

33. $\lim\limits_{u \to 1} \dfrac{u^4 - 1}{u^3 + 5u^2 - 6u}$

35. $\lim\limits_{s \to 16} \dfrac{4 - \sqrt{s}}{s - 16}$

37. $\lim\limits_{x \to 0} \dfrac{1 - \sqrt{1 - x^2}}{x}$

45. Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leqslant x < 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.

(i) $\lim\limits_{x \to 0^+} f(x)$ (ii) $\lim\limits_{x \to 0^-} f(x)$ (iii) $\lim\limits_{x \to 0} f(x)$

(iv) $\lim\limits_{x \to 3^-} f(x)$ (v) $\lim\limits_{x \to 3^+} f(x)$ (vi) $\lim\limits_{x \to 3} f(x)$

(b) Where is f discontinuous?

(c) Sketch the graph of f.

“Diagnostic Tests”

“Test A”

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}

(d) $\dfrac{5^{23}}{5^{21}}$ (e) $\left(\dfrac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$

(b) $(3a^3b^3)(4ab^2)^2$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$

(c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ (d) $(2x + 3)^2$

(e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$

(c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$

(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x + 3}{2x + 1}$

(c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x + 1}{x + 2}$ (d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

6. Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5}-2}$

(b) $\dfrac{\sqrt{4+h}-2}{h}$

7. Rewrite by completing the square.

(a) $x^2 + x + 1$

(b) $2x^2 - 12x + 11$

8. Solve the equation. (Find only the real solutions.)

(a) $x + 5 = 14 - \frac{1}{2}x$

(b) $\dfrac{2x}{x+1} = \dfrac{2x-1}{x}$

(c) $x^2 - x - 12 = 0$

(d) $2x^2 + 4x + 1 = 0$

(e) $x^4 - 3x^2 + 2 = 0$

(f) $3|x - 4| = 10$

(g) $2x(4-x)^{-1/2} - 3\sqrt{4-x} = 0$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4 < 5 - 3x \leqslant 17$

(b) $x^2 < 2x + 8$

(c) $x(x-1)(x+2) > 0$

(d) $|x - 4| < 3$

(e) $\dfrac{2x-3}{x+1} \leqslant 1$

10. State whether each equation is true or false.

(a) $(p+q)^2 = p^2 + q^2$

(b) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$

(c) $\sqrt{a^2 + b^2} = a + b$

(d) $\dfrac{1 + TC}{C} = 1 + T$

(e) $\dfrac{1}{x-y} = \dfrac{1}{x} - \dfrac{1}{y}$

(f) $\dfrac{1/x}{a/x - b/x} = \dfrac{1}{a-b}$

"Test B"

1. Find an equation for the line that passes through the point $(2, -5)$ and
   (a) has slope -3
   (b) is parallel to the x-axis
   (c) is parallel to the y-axis
   (d) is parallel to the line $2x - 4y = 3$

2. Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

4. Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
   (a) Find the slope of the line that contains A and B.
   (b) Find an equation of the line that passes through A and B. What are the intercepts?
   (c) Find the midpoint of the segment AB.
   (d) Find the length of the segment AB.
   (e) Find an equation of the perpendicular bisector of AB.
   (f) Find an equation of the circle for which AB is a diameter.

5. Sketch the region in the xy-plane defined by the equation or inequalities.
   (a) $-1 \leqslant y \leqslant 3$ (b) $|x| < 4$ and $|y| < 2$
   (c) $y < 1 - \frac{1}{2}x$ (d) $y \geqslant x^2 - 1$
   (e) $x^2 + y^2 < 4$ (f) $9x^2 + 16y^2 = 144$

"Test C"

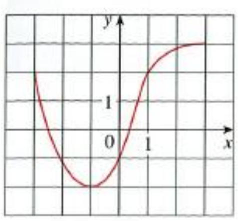


FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
   (a) State the value of $f(-1)$.
   (b) Estimate the value of $f(2)$.
   (c) For what values of x is $f(x) = 2$?
   (d) Estimate the values of x such that $f(x) = 0$.
   (e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2 + h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.
   (a) $f(x) = \dfrac{2x + 1}{x^2 + x - 2}$ (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$ (c) $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$

4. How are graphs of the functions obtained from the graph of f?
   (a) $y = -f(x)$ (b) $y = 2f(x) - 1$ (c) $y = f(x - 3) + 2$

5. Without using a calculator, make a rough sketch of the graph.
   (a) $y = x^3$ (b) $y = (x + 1)^3$ (c) $y = (x - 2)^3 + 3$
   (d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$
   (g) $y = -2^x$ (h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$
   (a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.
   (a) $f \circ g$ (b) $g \circ f$ (c) $g \circ g \circ g$

"Test D"

1. Convert from degrees to radians.

   (a) $300°$ (b) $-18°$

2. Convert from radians to degrees.

   (a) $5\pi/6$ (b) 2

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of $30°$.

4. Find the exact values.

   (a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$

5. Express the lengths a and b in the figure in terms of θ.

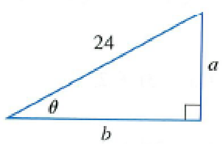


FIGURE FOR PROBLEM 5

6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

7. Prove the identities.

   (a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$

   (b) $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$

8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leqslant x \leqslant 2\pi$.

9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.